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ABSTRACT

This study examined the theory asserting that partitioning a unit is basic in developing understanding of the different rational number constructs. A fraction sequence was developed in which early experiences with partitioning units were provided. An alternative fraction sequence was designed to include initial activities with pattern blocks in which fractional parts of a region are covered by blocks instead of drawing lines or splitting sets. Both fraction sequences were taught for 2 weeks in two fourth-grade classes of mostly language minority students from low social-economic status (SES) families. A repeated-measures design was adopted using a 40-item instrument assessing 8 different fraction topics. In addition, three students from each class were individually interviewed. Analysis of covariance did not indicate that the experimental fraction curriculum was superior than the alternative one while results from the videotaped interviews indicated that students' fraction knowledge was incomplete and unstable during the 4-week period. The interviews revealed students' strategies in dividing units and in using concrete materials which clearly influenced their fraction ideas. It is concluded that the findings support the theory which views partitioning a unit as critical in building rational number concepts. Contains 17 references. (Author/JRH)

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UNIT PARTITIONING AS A MECHANISM FOR CONSTRUCTING BASIC FRACTION KNOWLEDGE: TESTING A HYPOTHESIS

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ABSTRACT

This study examined the theory asserting that partitioning a unit is basic in developing understanding of the different rational number constructs. The major research question states: Can a fraction teaching sequence based on unit partitioning result in a superior knowledge of six basic fraction concepts? A fraction sequence was developed in which early experiences with partitioning units (i.e., by drawing lines or forming subsets) were provided. An alternative fraction sequence was designed to include initial activities with pattern blocks in which fractional parts of a region are covered by blocks instead of drawing lines or splitting sets. Subsequent lessons followed the sequence in a regular fourth-grade textbook showing premarked units. Both fraction sequences were taught by the researcher for two weeks in two fourth-grade classes of mostly language minority students from low SES families. A repeated-measures design was adopted using a 40-item instrument assessing eight different fraction topics. Three students from each class were individually interviewed three times. Analysis of covariance did not indicate that the experimental fraction curriculum was superior than the alternative one. While both groups showed a modest gain from pre- to posttest, the mean scores were lower than expected. Moreover, results from the videotaped interviews indicated that students' fraction knowledge was incomplete and unstable during the four-week period. The interviews revealed students' strategies in dividing units and in using concrete materials which clearly influenced their fraction ideas. The findings support the theory which views partitioning a unit as critical in building rational number concepts.

Unit Partitioning as a Mechanism for Constructing Fraction Knowledge: Testing a Hypothesis[1]

Theoretical Background

The concept of a unit is critical as children try to make sense of fractional quantities and other mathematical topics (Case, 1988; Hiebert & Behr, 1988). Steffe and Cobb's (1988) seminal work on abstract composite/iterable units in the realm of counting numbers is directly related to students' development of rational numbers to the extent that whole numbers influence how students learn fractions. The unit in fractional situations poses additional cognitive demands as the nature of the unit changes. That is, units are partitioned and the number represents only a part of a unit (Hiebert & Behr, 1988). The research on rational numbers also indicates that unitizing (Lamon, 1994) as well as partitioning a unit (Kieren, 1976; Pothier & Sawada 1983) are both basic constructs in the development of rational numbers. Yet, previous studies (Behr, et al., 1983) have found that elementary students lacked partitioning skills needed to understand fraction concepts such as equivalence. It has also been reported that students' (Gr 4-8) partitioning strategies were influenced by social practices and the commodity being shared (Lamon, 1996). Moreover, increasing levels of partitioning strategies have been identified among children (ages 5-10) who were given tasks involving fair sharing within a social setting (Pothier & Sawada, 1983;1990). Partitioning is defined as the "act of dividing a quantity into a given number of parts which are themselves qualitatively equal (Kieren & Nelson, 1981, p. 39)".

Kieren's (1988) constructive model for knowing rational numbers defines subconstructs (partitioning, dividable units, equivalencing) along with a hierarchy of knowledge--from an intuitive, local view of rational number to the formation of conceptual fields for multiplication. Kieren's model raises questions as well as challenges for teaching rational numbers. What features of instruction would enhance such knowledge building? What is the nature of the curriculum which supports the development of basic fractions?

While there is agreement about the inherent complexities in understanding rational number, there is no consensus about how to facilitate the learning of rational numbers (Behr, et al., 1992). It has been suggested that identifying the kinds of experiences children need to develop meanings for rational numbers should be undertaken to inform curriculum and instruction.

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[1] Part of a dissertation study directed by Dr. Vicky Kouba, The State University of New York-Albany, 1993.

Purpose of Study

This study examined Kieren's (1976) hypothesis that partitioning a unit is a fundamental concept in the development of rational number concepts by comparing two alternative fraction sequences one of which was built on direct partitioning as a mechanism for developing basic fraction concepts.

Research Questions

The first question addressed in the study alluded to whether or not students could gain concepts of partitioning through direct instruction. The second question was stated as follows: If it is true that partitioning a unit is basic to the acquisition of a quantitative understanding of fractions, will a teaching sequence based on partitioning support the development of fraction understanding?

Methods and Procedures

Development of the experimental curriculum. A pilot study was initially conducted to empirically test whether test items that might involve partitioning a unit preceded items that assessed fraction concepts. A 30-item instrument was administered to two hundred and fifty-eight students (Gr 4-8) from south Texas. Results from the McNemar chi-square statistical tests indicated that items requiring partitioning skills preceded those items requiring fraction concepts like constructing the whole from a part. These results were also used in sequencing the fraction concepts in an experimental curriculum that initially provided partitioning activities by drawing lines or cutting (area model) and grouping (set model). The next step was the development of fraction concepts based on partitioning. The concept of "fair share" was emphasized as students engaged in physically dividing a whole into equal parts and observing relationships about the parts. Thus, the experimental curriculum focused on "direct" partitioning in order to build fraction ideas.

Development of an alternative fraction sequence. Another fraction curriculum was designed such that "indirect" partitioning of units was first introduced by asking students to cover completely continuous wholes with pattern blocks.This was then followed by instruction based on the fraction sequence presented in a traditional textbook in which premarked units were given. In both classes students were actively engaged in hands-on activities. However, students from the experimental class were encouraged to apply "direct" partitioning on area and set model to show part-whole relations and were not given pre-partitioned models. Moreover, pattern blocks were made available only to the students using the alternative fraction curriculum.

Development of assessment instruments. A forty-item paper-pencil assessment (test-retest r_{xy}=.83) of fraction concepts was developed by the investigator which measured students' understanding of the following: part-whole comparison, part-part relations, fraction terms, fraction of a unit, ordering, equivalence,and construct the whole from a part.

Description of sample. Two intact fourth-grade classes (n=40) from a suburban public school in south Texas were selected for the study. Both classes consisted of students who were mostly Limited English Proficient (LEP) from low SES families. Three students (high, average, low ability based on pretest score) from each class were selected to participate in three videotaped interviews scheduled before, during, and two weeks after the treatment.

Instruction. Both the experimental class and the class receiving the alternative curriculum were taught by the investigator for two weeks (10 hours). All sessions have been audiotaped and students' work documented and analyzed.

One-on-one interviews . Interviews were conducted to (a) trace students' understanding of fractions over time, (b) observe how partitioning influences student solutions to fraction tasks, and (c) gain insight into students' use of different representation systems for fractions as parts of a whole. During the one-on-one videotaped interviews, ten fractional tasks were presented in random order. The interviewees were asked to justify their answers by modeling the solution with concrete materials provided for them or marking on a pictorial model and verbalizing the process. In the last interview, follow-up questions were asked to probe further into students' thinking. All interviews and instruction were conducted in English.

Results

The first question addressed in the study alluded to whether or not students could gain concepts of partitioning through direct instruction. Evidence from the post- and retention tests showed that the experimental class had a modest gain from direct instruction on partitioning units (area and set) into "thirds", "fifths" and "sixths". A marked difference in partitioning ability between the two classes was observed when the unit was a regular polygon.

Comparing students' graphical solutions on the pretest with those on the posttest, it became apparent that the number of inappropriate partitioning strategies applied to area units decreased considerably in the experimental group. Such was not the case in the alternative treatment class that received instruction in basic fraction concepts

using prepartitioned units only. The students in this class were more successful than those in the experimental class in showing "sixths" on a rectangular model which is the dominant model in a traditional mathematics textbook.

The second question was stated as follows: If it is true that partitioning a unit is basic to the acquisition of a quantitative understanding of fractions, will a teaching sequence based on partitioning support the development of fraction knowledge? Pre-, post-, and retention tests, daily quizzes, audiotape lessons and teacher's daily journal were used to answer this question. Overall, no significant differences between the two treatments were found when CAT score was controlled. Students benefited from both fraction sequences as indicated by the significant gain in mean scores from pre- to post- to retention test. However, the mean scores on the last two tests for either class were still marginal. The experimental class (direct partitioning) showed more success than the alternative treatment class in representing fractions pictorially given an unmarked area model. Meanwhile, the videotaped interviews revealed the following:

- Despite emphasis during instruction on fraction terms (thirds, fourths, fifths), students' misconceptions prevailed particularly, when the set model was used.
- Students' understanding of fractions was unstable; Their whole number knowledge interfered in their ability to use partitioning strategies appropriately and consequently, the construction of a stable fraction knowledge was not facilitated.
- Students tended to use the part-part strategy in representing fractions more often when the unit showed unequal parts.
- Students used manipulatives to (a) justify incorrect answers, not as a tool to arrive at the expected answer, and (b) physically represent the numerator and the denominator of the fraction instead of using them to represent the unit and its parts.

Discussion

Although the difference in fraction achievement between the two treatments was not statistically significant, findings from this study supported Kieren's hypothesis for knowing rational number in lieu of evidence indicating partitioning (cutting, grouping, separating) units is a constructive tool that enhance understanding of fractions. While the findings in the present study are limited to the part-whole interpretation of rational number, the evidence gathered indicated that, through direct partitioning activities, fourth-grade students showed improvement in (a) naming equivalent fractions, (b) associating a fraction with an area model that was not pre-partitioned, and c) comparing two proper fractions. The benefits of the experimental curriculum were

manifested more in students' solutions to problems involving an area model than a set model indicating an improvement in spatial reasoning.

For some students in the experimental class, teaching fraction concepts based on partitioning was not quite successful in orienting them to the equal-part condition associated with the part-whole interpretation of fractions. A student (Carla) was able to attend to the equal-part condition when she had to partition the whole (pictorial) prior to representing the fraction in question. But when she was shown a pictorial area model that had unequal divisions, she immediately counted the parts "shaded" and the parts "unshaded".

It is possible that students in the experimental class could have developed misconceptions during instruction which were revealed during the interviews. For instance, students seemed to think that fractions require equal parts only when the unit is not divided but not when the whole is already divided in some way. Students appeared to use double lenses in viewing fractions.

Conclusion and Significance of Study

Due to the limitations of the study, no conclusive statement can be made from the data. This exploratory study however, made possible the formulation of research hypotheses which can further advance our knowledge about the teaching and learning of fractions and about rational number in general. This study provided a new perspective to Kieren's conceptualization of the kinds of knowledge needed to develop rational number concepts and has gathered substantial evidence about the nature of student understanding over time. Findings from this investigation have contributed to the ongoing debate about the appropriate role and relative merits of instructional materials in the mathematics classroom (see Ball, 1992). More importantly, this study provided teachers with an alternative curriculum that addresses not only prerequisite knowledge and skills which support the acquisition of fraction concepts but also the learner's intuitive knowledge base.

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